KAP’s Corner

FEA Thermal Strain

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The Cosmos family of FEA tools features a user-interface that is so easy to learn, we sometimes have trouble convincing people of the necessity for formal training. I feel that sales literature and tutorial examples err on the side of over-simplifying the task of engineering analysis. Certainly, the friendly Cosmos interface makes the button-pushing aspect of FEA very easy to apprehend – but analysis is more than pushing software buttons. I sometimes get calls from frustrated analysis users who have fallen afoul of some foible that does not satisfy their intuition. One such recurring Hotline call is from users who are trying to extend their thermal analysis into the realm of non-linear material response, particularly the Coefficient of Thermal Expansion, (C-T-E).

My goal here is mercenary. In the future, I can shorten the duration of this type of hotline call by saying, “refer to the Kap’s Corner article”. But I also present this article in hopes it not only cure, but also prevent, you the user from experiencing thermally induced Stress and Strain. If you wish to extend your thermal studies into materials with non-linear thermal expansion, this KAP’s Corner is for you.

About KAP

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Keith Pedersen has a BSME from Clarkson College and an MSME from Boston University. After a stint at General Electric in Burlington, VT, Keith was the lead Applications Engineer for Advanced Surfacing products for Matra Datavision USA, including EUCLID-IS, UniSurf, and STRIM. He joined CAPINC in 1998 to support advanced surfacing applications in SDRC I-DEAS and joined our SolidWorks group one year later. Keith has extensive industry and consulting experience in non-linear Finite Element Analysis and Computational Fluid Dynamics in addition to surfacing applications. He is a Certified SolidWorks Professional (CSWP) and certified to train and support COSMOSWorks.

About CAPINC

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BASICS: The Coefficient of Thermal Expansion (CTE)

Let's first establish some common ground concerning thermally induced strain, and stress. In a linear FEA analysis that includes temperature changes, you are required to fill in the COSMOS material property ALPX, which is the Thermal Expansion Coefficient. This is usually referred to in industry and college texts as "CTE", and I'll stick to that name here. This is a measure of how much the material will grow (or shrink) in length, with a change in temperature. It has units of 1/degree.

Aluminum, for example, has a CTE of about 0.000024 / degrees Kelvin. The coefficient does not have, (or need), an included unit of length, because the units will cancel out in application. You can imagine, if you wish, that units of length appear in both the numerator AND the denominator, and the CTE value will be true for any system of length units:

C-T-E (Al) = 0.000024 inches / (1 inch * 1 °K) , Or;

C-T-E (Al) = 0.000024 meters / (1 meter * 1 °K)

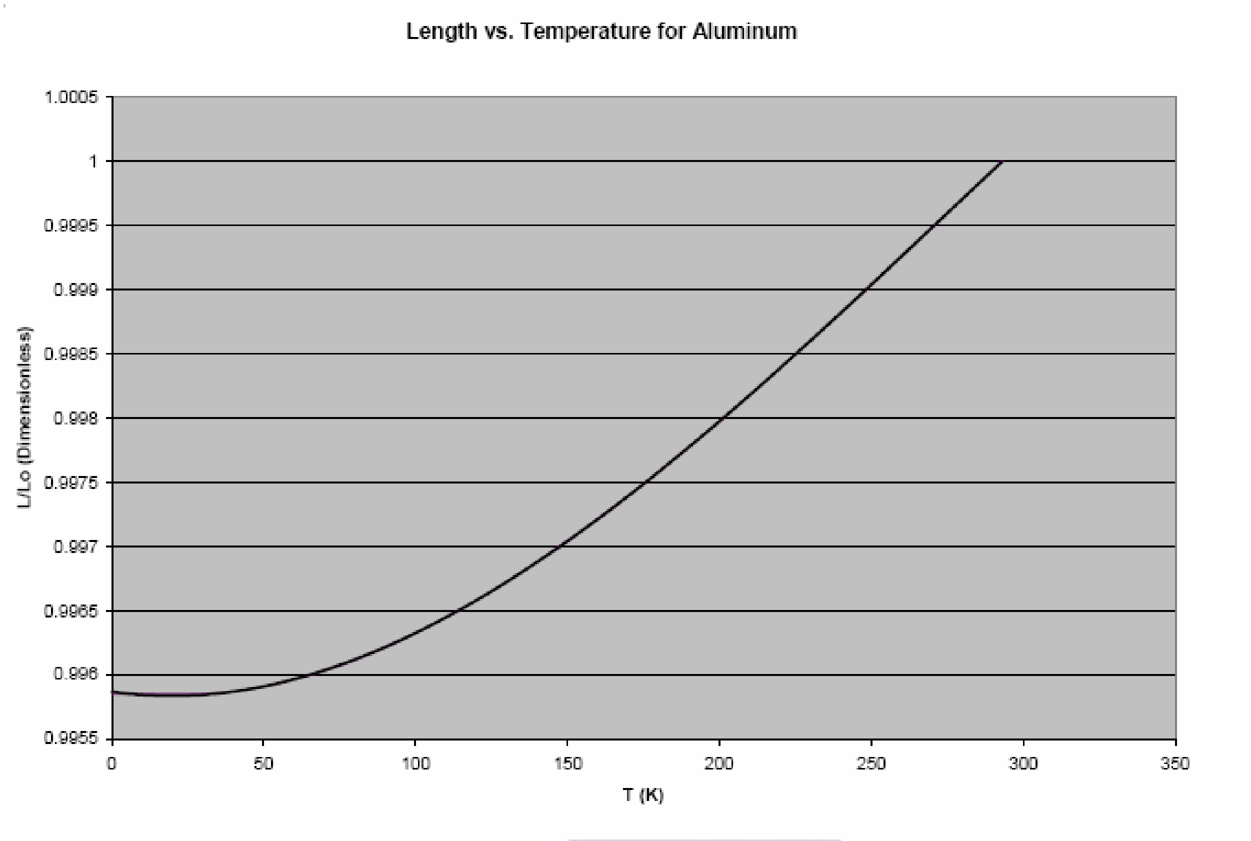
Both statements are true. To calculate the amount of growth (or shrinkage) of a material sample, you multiply this factor, by the temperature change, and also by the original length of the sample part. If a bar of aluminum were 100" long, for example, and were subjected to a 10°K temperature rise, the growth would be;

Strain = (0.000024 / °K) x 100 inches x 10°K = 0.024 inches

Cosmos performs this simple calculation at each finite element, and adds that strain into the equations to find the state of stress where the element is in equilibrium with your other loads and boundary conditions.

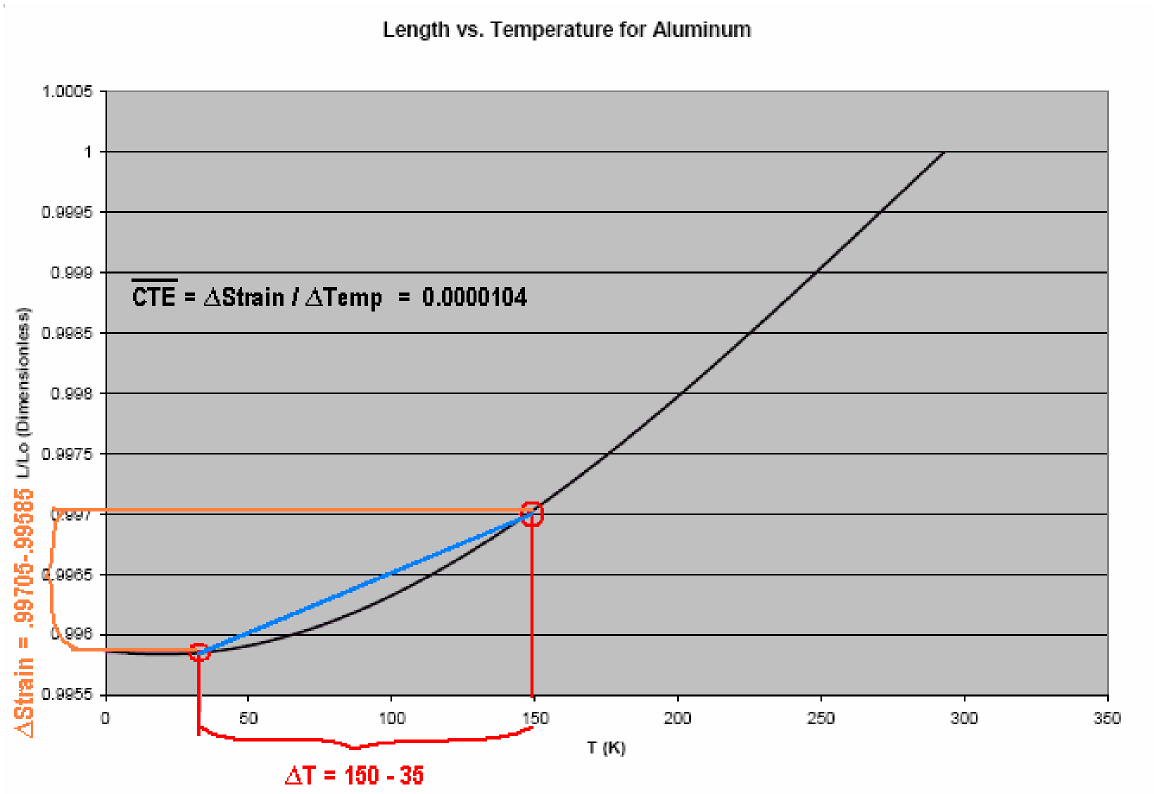
REALITY: Linear CTE is Relative, not Absolute

O.K., the application of a single, scalar value of CTE for a material carries with it a whole lot of assumptions. Below you see a chart of the real-world behavior of a sample of aluminum. Assuming the bar is 1" long when measured at 293°K, this chart shows the measured length of the bar as it is cooled towards absolute zero, (from the website of Columbia University; their data drawn from the National Bureau of Standards). Clearly, a single, scalar value for thermal strain cannot accurately describe all this behavior.



We can apply a scalar CTE in two ways. When you look up a CTE value in a handbook, or on MatWeb.com, etc., it is almost certainly a relative value of CTE, and is only valid within a range of temperatures. In the graph above, we can see that the curve looks nearly linear from 250°K out to 300°K. If we were to measure the slope of this line, we would obtain a value very close to the one I quoted above, about .000025 / °K. For a Linear FEA analysis, this is what CosmosWorks assumes your CTE to be – a measure of the slope of the material strain curve. Note, it does not care at all about the height of the line – where the Y-intercept would be, for example – because all we need to know is the relative change in strain between a start and an end temperature.

So, what if you need to analyze the behavior of a aluminum part that is chilled from, say, 150°K down to 30°K? We can see that the behavior in this region is not linear. You can still perform a linear FEA analysis, if you first hand-compute an absolute value of CTE between these two temperatures. The figure below shows how you would pick off the start and end temperatures from the Strain curve, and calculate a CTE value that is valid just for that temperature interval. (Remember that a linear FEA problem only cares about the initial and the final, steady-state condition, and does not care at all about the path taken in between!).



Button Pushing the Linear Problem

In either of the two situations described above, we have assumed that the model begins the analysis at a uniform starting temperature, and the analysis concludes with the part(s) at a uniform ending temperature.

While you are in the LOAD/RESTRAINT – TEMPERATURE dialog box, you will notice that there are two radio buttons at the top, one for TEMPERATURE, and one for INITIAL TEMPERATURE. The latter of these is grayed-out - you cannot select it. This is because we are running a linear study. The assumption here is that the Initial temperature and the Zero-Strain Reference temperature are the same. To set the Zero-Strain reference temperature, right-click over the icon for the Study in the Feature Manager. Select PROPERTIES. Then, click the tab for FLOW/THERMAL EFFECTS. Enter the initial temperature at the start of the study here.

COSMOS can set the Initial temp. to be equal to our input zero-strain temp. for the following reason: If our CTE is a relative value, then it is assumed valid over a range of temperatures, and we don't care about the actual height of the CTE point – only its slope. So an absolute point-of-reference is not needed. If, on the other hand, we are using a CTE that was hand- calculated across two known temperatures, then that CTE is ONLY valid assuming that the first of those temperatures is also the zero-strain point of reference.

Both of these means of applying CTE are slope-based calculations, and in the literature you usually see them both represented with an overbar. However, when the CTE is valid only for a computed temp difference, and not across an entire range, I prefer to also include the reference temperature as a subscript, as shown at left.

$\overline{CTE}$ = Slope of the strain curve, valid across a range of temps

$\overline{CTE}_{tempX}$ = Slope of the strain curve, computed from a reference temp 'X'

The Non-Linear CTE Problem

Hopefully, most of the preceding discussion contained few surprises. Now let's investigate a thermal analysis where an assembly of parts comes to a steady-state temperature solution that is NON-uniform. Not only do the parts equilibrate at different temperatures, we can assume there are thermal gradients inside each part. What will the thermally-induced stresses be now?

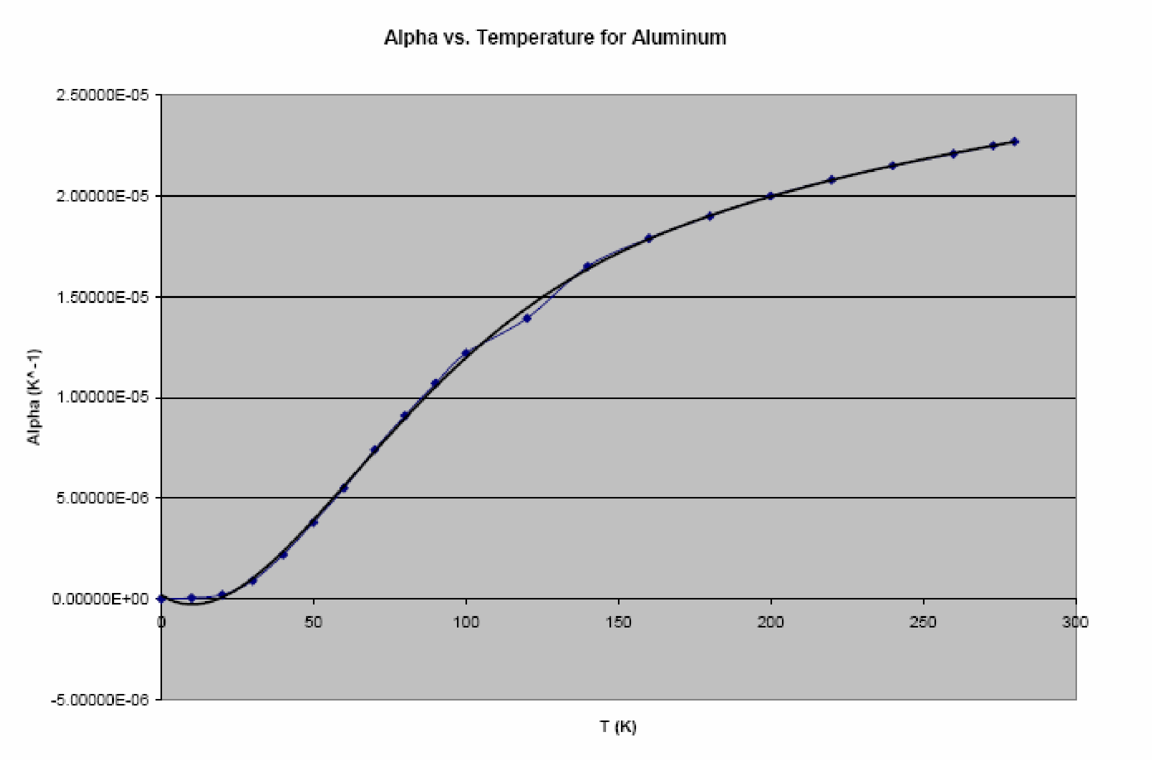
To solve this problem, we can no longer rely on an averaged value of CTE. We need to feed Cosmos a graph of CTE-vs-Temperature, so that at each node, the system can compute the local strain induced by that node's temperature. But this is where we can run afoul of the different ways of measuring and presenting CTE data. This is the source of the problem that most commonly offends the user's intuition – how it is possible to enter a 'wrong' graph of CTE. Tables of data on Thermal Strain can be presented in one of three forms, and it is very important that we be able to recognize which type of data we are looking at, before we start plugging values into Cosmos.

The Evolution of C-T-E Data

It all starts with Strain. There is no way in a lab to measure instantaneous, (that is, slope-based) values of CTE. Instead, you have to take measurements of Strain data, and then derive from there. The primary data collection yields a table of Length vs Temperature. The two graphs I've presented so far are drawn from L vs T data, where the Length has been converted to Strain, (the change in length divided by the original length).

Alpha (ά): It's like plotting all the values of slope-base CTE

The most intuitive way to present this data is to compute from it another table of the instantaneous, or local, values of CTE. This type of CTE is often referred to as "alpha", or via the symbol ά. At each temperature, you compute (or estimate) the slope of the strain curve. This is good for human interpretation, because you can visualize alpha as a 'spring rate', the stiffness of a thermal spring. At each temp. on the graph, imagine a very small change in temperature, say 1 degree either way – this would be the rate a sample would shrink or grow from that temperature. Tables and graphs in this form can be found in the appendices of most material science texts. The graph below is instantaneous, or local, CTE data (alpha) drawn from the strain data we have already looked at. Compare this graph below, to the prior one – they are saying the same thing, but two different ways.



Although graphing ά is a great communication and teaching tool, it is not in a form that is conducive to engineering, and especially to FEA. Why? First: the calculation of the local slope involves approximation, usually by some finite-difference method, or sometimes by trying to fit a (least squares?) curve to portions of the data, and then taking the local derivative of that curve's polynomial. Then: To apply this data to an

Alpha Graphs:

Pretty to look at. Not good for supporting strain calculations

analysis, we actually have to perform the reverse-approximation, Integrating the values of local strain each across their temperature range, to compute an aggregate deflection. Both the calculation of the slopes, and the re-integration of these slopes to accumulate the strain, present a great deal of round-off and truncation error. Programming a computer to do this for you would relieve the drudgery, but would not address the loss of accuracy.

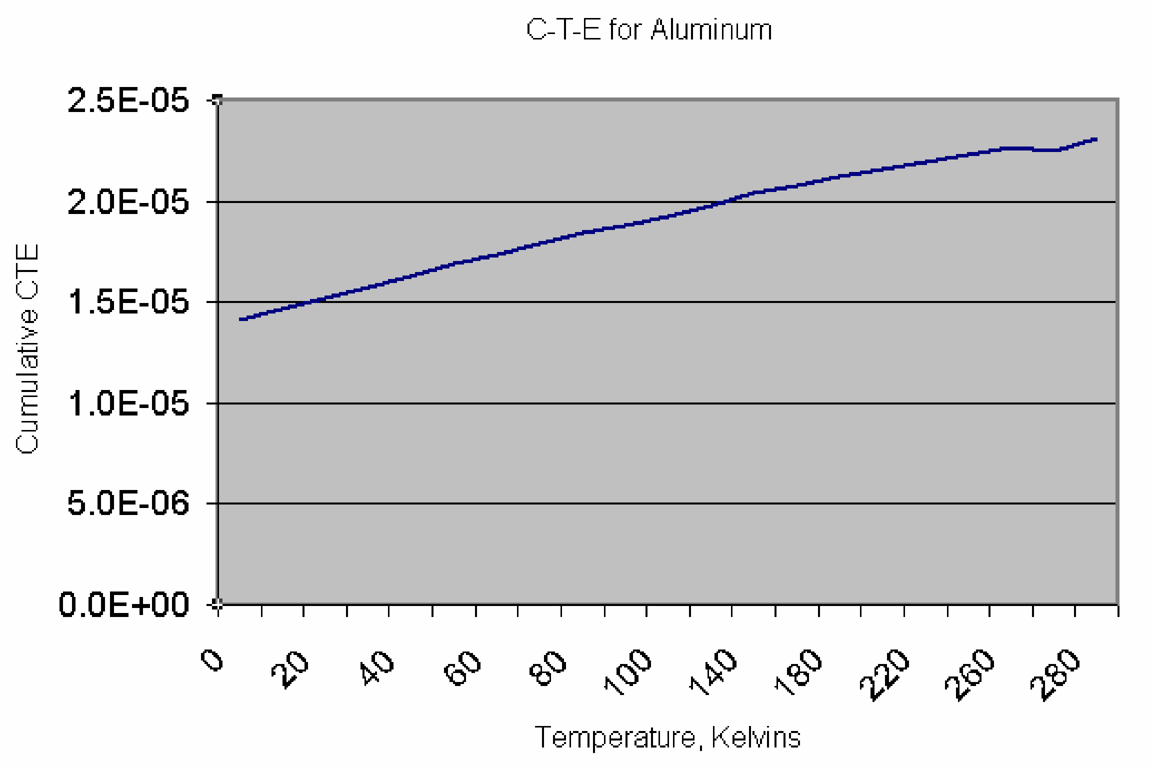
Fortunately, there is another way of presenting CTE data that does not require all these cumulative calculations.

Since the L-vs-T data is always measured on a sample that starts from a known length while at a known temperature, it makes sense to use that temperature as a reference baseline. So the third way of formatting L-vs-T data is to plot an absolute CTE relative to the reference temperature. Let us assume a reference temperature of 300K – then at each temperature T, this CTE would have the value;

$\mathrm{CTE}\{T\} = (\Delta L / L) / \Delta T$, or;

$\mathrm{CTE}\{T\} = (L\{300\} - L\{T\}) / L\{300\} / (300 - T)$

Another way of saying this is that these values of CTE are cumulative, embodying the net strain over the net temperature change, from the baseline temperature. If we plot CTE using this method for the same aluminum sample pictured above, the graph looks like this:



Input only graphs of Absolute CTE (relative to a reference temp.) in COSMOS

Notice how very different the graph of cumulative CTE is from the instantaneous (ά) CTE graph before it. The two graphs will usually only agree with each other in the immediate vicinity of the Reference Temperature. It is crucially important to input the right kind of CTE data for a Cosmos thermal study. Any non-linear CTE graph data you input has to be in this latter, cumulative format, for the FEA to return correct results.

To make use of this data in a Linear study, you must change the type of MATERIAL to "Custom Defined". In a custom material, you can change the ALPX coefficient from "Constant" to "Temp Dependant", at which point you will be able to fill in the table that defines the CTE graph.

Summary

When you are doing a linear FEA analysis that includes the effects of thermal strain, you input a single, scalar value for "ALPX", which is the Cosmos variable for the Coefficient of Thermal Expansion, (CTE). This value is a rate, assumed to be valid over the range of temperatures encountered, and so it is a measure of the slope of the Strain-vs-Temp graph. BUT – when you are doing a non-linear FEA, including the effects of non-linear thermal expansion, you must input a graph of Strain-vs-Temp, and this graph must show cumulative strain, all measured from a zero-strain reference temperature – it must NOT be a graph of instantaneous, or slope-based CTE. This graph is not what you frequently find in the appendix of college textbooks on material science; but it IS the format in which the data are originally collected in the lab, requiring far fewer calculations.

Have a SolidWorks bone to pick? Want more tips in a specific area of the CAD? Keith is looking for requests from users for future KAP Corner topics. Email your suggestions to: support@capinc.com, attn to Keith Pedersen.